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SUPRATHERMAL PROTON BREMSSTRAHLUNG BY THE WEIZSÄCKER-WILLIAMS METHOD

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Abstract

The Weizsäcker - Williams method for calculating the bremsstrahlung cross section utilizes the electron rest frame as the natural frame in which to carry out the calculation in terms of compton scattering the virtual photons of the proton electromagnetic field. Since suprathermal proton bremsstrahlung is a process in which the electron is initially at rest and the proton is moving the Weizsäcker - Williams method is obviously the most natural one to use in this case. In the present paper the cross section for suprathermal proton bremsstrahlung is calculated by means of this method and the results are shown to differ considerably from those of a previous paper in which the conventional and much more difficult method was used. In particular we conclude that the suprathermal proton bremsstrahlung process cannot explain the 2-6 Mev gamma rays reported by Vette _et al_.

I Introduction

The process of suprathermal proton bremsstrahlung (SPB) has been recently discussed (Boldt and Serlemitsos 1969; Hayakawa 1970; Brown 1970 a,b) as a possible source of cosmic X and γ rays. This process differs from ordinary bremsstrahlung only in that in this case the electron is at rest and the heavy particle (proton) is moving. It is therefore just ordinary electron-proton bremsstrahlung viewed from the electrons rest frame rather than from the protons rest frame as is usually the case.

The usual method of calculating the radiation cross section for SPB has been to view the process in the protons rest frame, treat it there as ordinary bremsstrahlung, and then transform back to the electron rest frame (lab frame). In the case of very non-relativistic protons ($\beta \ll 1$) the effect of the transformation back to the lab frame on the energy of the emmitted photons is insignificant and since the cross section is an invariant the result is just the Bethe - Heitler (Heitler 1954) formula in the non-relativistic approximation (Boldt and Serlemitsos 1969; Hayakawa 1970; Brown 1970a). In this formula the kinetic energy T must be interpreted as the kinetic energy of the electron in the protons rest frame ie. $T = (m_e/M_p)\, T_p$.

In the case where the proton is relativistic ($\beta \approx 1$), however, the Doppler shift of the photon frequency is important and must be taken into account. In treating this case (Brown 1970b) one must use a bremsstrahlung cross section that includes the angle of emission of the photon (Gluckstern and Hull 1953) since these enter into the transformation to the lab frame in a vital way.

Brown (1970b) has employed this method to compute a possible source of 1-100 Mev gamma rays that might explain the observations of Vette <u>et</u> <u>al</u> (1970) in this energy region. He derives a spectrum with a peak at ~ 5.5 Mev which when suitably red shifted can presumably fit the observations.

The cross section of Gluckstern and Hull (1953) and the necessary Lorentz transformation employed by Brown are quite complicated and tedious. It is the purpose of this note to point out that a far simpler method for calculating the SPB cross section exists and when it is employed one obtains spectra that are considerably different from those obtained by Brown(1970b).

It has long been known that one can obtain the Bethe - Heitler formulas for bremsstrahlung by means of the Weizsäcker - Williams method (Weizsäcker 1934; Williams 1935; Heitler 1954; Jackson 1962). In this approach the collision process is viewed from the rest frame of the electron. In this frame the electromagnetic field of the moving proton resembles a plane pulse of radiation. This pulse may be Fourier analyzed and the resulting photon spectrum is allowed to Compton scatter off of the electron making use of the Klein - Nishina cross section. The resulting scattered photon spectrum is then Lorentz transformed back to the proton rest frame and one obtains a spectrum that is in complete agreement with the Bethe - Heitler formulas for bremsstrahlung.

In this approach the simplest view of the process is obtained in the rest frame of the electron and since this is the natural lab frame for the SPB process the Weizsäcker - Williams method is obviously the natural one to use. Using this approach in calculating the SPB radiation cross - section

there is no need to keep track of angles and there are no Lorentz transformations required.

II The SPB Cross Section

From Jackson (1962) we have the number spectrum of virtual photons in the protons electromagnetic field given by

$$N(\alpha_o)\, d\alpha_o = \frac{2}{\pi} \frac{e^2}{\hbar c} \frac{1}{\beta^2} \ln(A/\alpha_o) \frac{d\alpha_o}{\alpha_o} \qquad (1)$$

where we have already integrated over all possible impact parameters and where $\alpha_o = \hbar\omega_o/m_e c^2$

and $A = 1.123\ (M_p/m_e)\ \gamma\ \beta^2\ \exp(-\beta^2/2)$.

The spectrum, equation(1) is non-zero only for $\alpha_o < A$; actually for $\alpha_o \approx A$ equation(1) is not strictly correct and the spectrum goes to zero exponentially as $\exp(-2\alpha_o m_e/M_p\ \gamma\beta^2)$.

This part of the spectrum does not contribute significantly to the scattered spectrum due to the drop of of the Klein - Nishina cross section at high energies, therefore we shall ignore this complication and use equation(1) as though it were exact.

The Klein - Nishina cross section for scattering a photon of energy α_o into an energy between α and $\alpha + d\alpha$ is given by

$$\frac{d\sigma}{d\alpha}(\alpha,\alpha_o) = \frac{\pi r_o^2}{\alpha_o^2}\left[\frac{1}{\alpha_o^2} + \frac{(2 + \alpha - 2/\alpha)}{\alpha_o} + \frac{(1 - 2\alpha)}{\alpha^2} + \frac{\alpha_o}{\alpha}\right] \qquad (2)$$

where r_o is the classical electron radius $e^2/m_e c^2$ and α has the constraint $\alpha_o \geq \alpha \geq \alpha_o/(1+2\alpha_o)$.

The SPB cross section is now obtained by a simple integration

$$\frac{d\sigma}{d\alpha}(\alpha) = \int_{\alpha}^{\alpha_u} \frac{d\sigma}{d\alpha}(\alpha,\alpha_o)\, N(\alpha_o)\, d\alpha_o \tag{3}$$

where $\alpha_u = \text{Min.}\ (A,\ \alpha/(1-2\alpha))$.

The integral in equation 3 may be performed in a straightforward, analytical manner to obtain, after some algebraic rearrangement

$$\frac{d\sigma}{d\alpha}(\alpha) = \frac{2r_o^2}{137}\left[f_1(\alpha) - f_2(\alpha)\right] \tag{4}$$

where

$$f_1(\alpha) = \frac{(\frac{1}{4}-\alpha + 4\alpha^2)}{3\alpha^4}\ \ln\ (A/\alpha)$$

$$-\ \frac{(13/8) - 5\ \alpha + 20\alpha^2}{18\alpha^4} \tag{5}$$

and if $\alpha < A/(1+2A)$

$$f_2(\alpha) = \frac{(\frac{1}{4}-\alpha+4\alpha^2-8\alpha^3+2\alpha^4-8\alpha^5)}{3\alpha^4}\ \ln\left[A(1-2\alpha)/\alpha\right]$$

$$-\ \frac{(13/8)-8\alpha + 29\alpha^2 - 40\alpha^3 + 10\alpha^4 - 16\alpha^5}{18\alpha^4} \tag{6}$$

and if $\alpha > A/(1+2A)$

$$f2(\alpha) = -\frac{1}{16A^4} + \frac{(2\alpha+\alpha^2-2)}{9A^3\alpha} + \frac{(1-2\alpha)}{4A^2\alpha^2} + \frac{1}{A\alpha}\ . \tag{7}$$

Equations (5) through (7) appear a bit complicated yet may be computed and plotted quite simply. In Figure 1 we show curves of $(d\sigma/d\alpha)\ (2r_o^2/137)^{-1} = f_1(\alpha) - f_2(\alpha)$ plotted for various values of $\gamma = E_p/M_pc^2$ ranging from $\gamma = 1.1$ to 40.

III Discussion

If one converts the scale in Figure 1 to cm^2/Mev $(2r_o^2/137 = 1.16 \times 10^{-27}cm^2$; a unit of α is 0.511 Mev) one can compare these results with those of Figure 1 of Brown (1970b).

For γ=10~20 the cross sections are comparable at α=10Kev but the present curves are about two orders of magnatude below those of Brown at α = 1 Mev. Browns curves are roughly proportional to α^{-1} while the present results tend to be steeper than α^{-2} at least for $\alpha>\frac{1}{2}$. Also there appears to be no maximum in the cross section for a proton energy of 10 Gev as was reported by Brown (1970b). It is difficult to imagine how such a maximum could arise in this problem because there is no characteristic energy of this order in the electromagnetic interaction of electrons and protons.

If one examines the curves of Figure 1 for $\alpha>\frac{1}{2}$ (255 Kev) (the slight bump in the curves at $\alpha\approx\frac{1}{2}$ is due to the transition from equation (6) to equation (7); presumably using the more correct form for $N(\alpha_o)$ would have smoothed this out some) one notices that the slopes are never any flatter than $\alpha^{-\Gamma}$ with $\Gamma\approx 2.25$. Therefore, no distribution of proton energies can produce a spectrum of photons that is flatter than this for energies above

255 kev. Including a cosmological red shift will not alter this argument, thus it is difficult to see how the SPB process can produce the flattening of the spectrum to $\Gamma<2$ reported by Vette et al (1970). Incidentally, the above argument applies equally well to the cross section curves of Brown (1970b) and it is therefore difficult to see how he obtains a positive slope and the maximum in his final spectrum.

All in all it appears dubious that the SPB process can explain the gamma rays reported by Vette et al (1970) but in those cases where the SPB cross section is relevant the Weizsäcker - Williams method is the simplest and most natural way to calculate it.

References

Boldt, E. and Serlemitsos, P. 1969, Ap. J. 157, 557

Brown, R. L. 1970 a, Ap. J. (Letters), 159, L187

____________ 1970b, Lettere al Nuovo Cimento, 4, 941

Gluckstern, R. L. and Hull, M.H.Jr. 1953, Phys. Rev., 90, 1030

Hayakawa, S. 1970, Symp. No. 37 of I.A.U. on Non-Solar Gamma and X-Ray Astronomy, Rome, May 10, 1969 (Dordrecht: D. Reidel Publ. Co.)

Heitler, W. 1954, The Quantum Theory of Radiation (3d ed.; New York: Oxford University Press) Section 25, Appendix 6

Jackson, J. D. 1962, Classical Electrodynamics, (New York: John Wiley and Sons, Inc.) Sections 15.5, 15.6

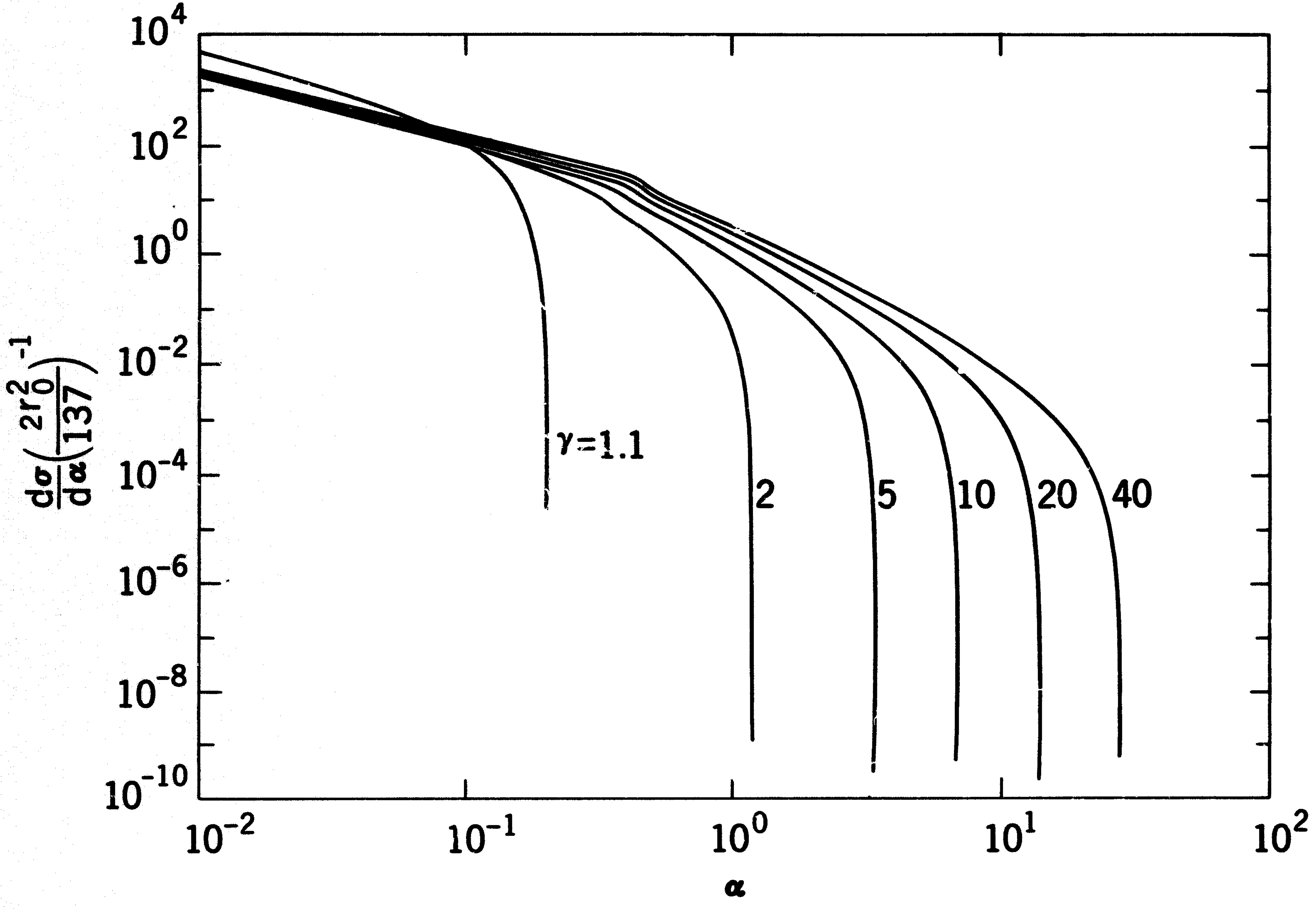
Vette, J. I., Gruber, D., Matteson, J. L. and Peterson, L. E. 1970, Ap. J. (Letters), 160, L161

Weizsäcker, C. F. v. 1934, Z. Phys., 88, 612

Williams, E. J. 1935, Kgl. Dansk. Vid. Selsk. Mat. fys. Medd., 13, No. 4

Figure Captions

Figure 1 - Plot of the suprathermal proton bremsstrahlung (SPB) Cross section as a function of $\alpha = \hbar\omega/m_e c^2$ for various values of $\gamma = E_p/M_p c^2$. The cross section is plotted in units of $2r_o^2/137$ ($=1.16 \times 10^{-27} cm^2$) per unit α.



10^4
10^2
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$\frac{d\sigma}{d\alpha}\left(\frac{2r_0^2}{137}\right)^{-1}$
$\gamma=1.1$
2
5
10
20
40
10^{-2}
10^{-1}
10^0
10^1
10^2
α